ANALYSIS

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Constructions of diophantine quadruple with property $D\ (6\ pq)^2$

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ABSTRACT

This paper concerns with the study of constructing non-zero integer quadruple (a,b,c,d) such that the product of any two elements of the set increased by a square is a perfect square. Different relations between the elements of the quadruple and special numbers are presented.

Keywords: Diophantine Quadruple, System of Equations.

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1. INTRODUCTION

The problem of the construction of sets with property that product of any two of its distinct elements is one less than a square has a very long history and such sets were studied by Diophantus(Bashmakova .I.G. (1974). For an extensive review of various article one

may refer Thamotherampillai.N (1980); Brown.E.(1985); Gupta.H, and Singh.K,(1985); Beardon.A.F and Deshpande.M.N (2002); Deshpande.M.N, (2002,2003); Bugeaud Y.,Dujella.A, and Mignotte (2007), Tao Liqun (2007); Fujita Y. (2008), .Srividhya.G (2009); Gopolan.M.A, (2011,2012); Yasutsugu Fujita, Alain Togbe (2011). In this paper, starting with the diophantine pair (a,b) with the property $D(s^2)$, we extend it to diophantine triple with property $D(s^2)$ and quadruple with property $D(6pq)^2$.

2. NOTATION

P_n^m : Pyramid number of rank n with size m
$T_{m,n}$: Polygonal number of rank n with size m
SO_n: Stella octangular number of rank m
Pr_n: Pronic number of rank n
OH_n: Octahedral number of rank n
Pt_n: Pentatope number of rank n

3. METHOD OF ANALYSIS

Let $a = r - s$, $b = r + s$ where, r and s are non-zero distinct integers and the product ab is square free, be any two non-zero integers. Observe that (a,b) is a diophantine double with property $D(s^2)$.

Let c be any non-zero integer such that

$$ac + s^2 = \alpha^2 \tag{1}$$

$$bc + s^2 = \beta^2 \tag{2}$$

Eliminating c between (1) and (2) we get

$$b\alpha^2 - a\beta^2 = (b-a)s^2$$

The choice $\alpha = X + aT$, $\beta = X + bT$ (3)

leads the above equation to the Pell equation

$$X^2 = abT^2 + s^2 \tag{4}$$

whose initial solution is

$$T_0 = 1,\quad X_0 = r \tag{5}$$

Using (5) in (3) and employing either (1) or (2) we get

$$c = 4r$$

Thus, $(r - s, r + s, 4r)$ is a Diophantine triple with the property $D(s^2)$.

The triple can be extended to a quadruple as follows .
Let d be any non-zero integer such that

$$ad + s^2 = \overline{\alpha}^2 \tag{6}$$

$$bd + s^2 = \overline{\beta}^2 \tag{7}$$

$$cd + s^2 = \overline{\gamma}^2 \tag{8}$$

Eliminating d between (7) and (8) w get

$$c\overline{\beta}^2 - b\overline{\gamma}^2 = s^2(c-b) \tag{9}$$

Taking the linear transformations

$$\overline{\beta} = X + bT \qquad \overline{\gamma} = X + cT \tag{10}$$

in (9) it becomes

$$X^2 = bcT^2 + s^2 \tag{11}$$

whose initial solution is $T_0 = 1$, $X_0 = \beta$

Substituting the above values in (10) and employing (7) we get

$$d = 9r + 3s \tag{12}$$

Using (12) in (6) and simplifying we have

$$(3r - s)^2 = 3s^2 + \overline{\alpha}^2 \tag{13}$$

which is satisfied by

$$s = 2pq, \; r = \frac{3p^2 + q^2 + 2pq}{3}, \quad \overline{\alpha} = 3p^2 - q^2 \tag{14}$$

Since our thrust is on integers, note that r is an integer when q is replaced by $3q$ .Thus,

$$r = p^2 + 3q^2 + 2pq$$
$$s = 6pq$$
$$d = 9p^2 + 27q^2 + 36pq$$

Therefore we obtain $(p^2 + 3q^2 - 4pq, p^2 + 3q^2 + 8pq, 4p^2 + 12q^2 + 8pq, 9p^2 + 27q^2 + 36pq)$ as a diophantine quadraple with the property $D(6pq)^2$.

Some numerical examples are presented below

p	q	(a, b,c,d)	property $D(6pq)^2$
2	3	(7,79, 172,495)	$D(36)^2$
2	4	(20, 116, 272,756)	$D(48)^2$
3	4	(9, 153, 324,945)	$D(72)^2$
3	5	(24, 204,456,1296)	$D(90)^2$
2	5	(39,159,396,1071)	$D(60)^2$

Denoting a, b, c, d by $a(p,q)$, $b(p,q)$, $c(p,q)$, $d(p,q)$ respectively the following relations are observed.

- $\mathbf{2b(p^2, q^2) - 2a(p^2, q^2) = 6(4p^2q^2)}$ is a nasty number.

- $9b(p^2,q^2)-d(p^2,q^2)=6(6p^2q^2)$ **is a nasty number.**
- $a(p,2p)-t_{8,p}+2p=0$
- $a(n,n+1)+4p_{r_n}-t_{10,n}\equiv 0(\bmod 3)$
- $b(n,2n^2+1)-24OH_n-12t_{4,n^2}-t_{28,n}\equiv 0(\bmod 3)$
- $b(n,2n^2-1)-8SO_n-12t_{4,n^2}+t_{24,n}\equiv 3(\bmod 11)$
- $c(n^2,n+1)-16P_n^5-4t_{4,n^2}-t_{26,n}\equiv 12(\bmod 13)$
- $b(n(n+1),(n+2)(n+3))-a(n(n+1),(n+2)(n+3))-288Pt_n=0$
- $9b(n(n+1),(n+2)-d(n(n+1),(n+2)-216P_n^3=0$
- $4a(n,2n^2-1)-c(n,2n^2-1)+12SO_n=0$
- $9a(n,n+1)-d(n,n+1)+72\Pr_n=0$
- $a(p,p)=0$
- $b(n^2,n+1)-c(n^2,n+1)=48P_n^5$

4. CONCLUTION

In the construction of the quadruple we have assumed the product a b is square free. One may assume that the product a b is a perfect square and search for Diophantine quadraples with suitable property.

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